On Inverse Analytic Methods and Monotonous Growth of Knowledge

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Abstract:

In this paper, I briefly present the instructional model of method and the notion of an analytic method. Then, based on a case study, I discuss whether the notion of monotonous growth of portions of states of knowledge, which is presupposed in the formalization of the instructional model of method, is well established. I argue against the dependence of the notion of analytic method on the notion of methods which necessarily provide the growth of explicit knowledge of agents. The main reason for my claims is the existence of methods of reduction. Based on this result, I further question the possible existence of inverse analytic methods, which would function as reversals of other analytic methods. I deny the existence of such methods in general.

Keywords: method, instruction, knowledge state, reduction, monotonous growth

Introduction

In our model of scientific methods (Bielik et al. 2014a,b,c,d),[1] a method is modeled as an ordered set of instructions for actions that aim to solve a problem.[2] A scientific method is simply a method used in science to solve some problem of actual cognitive interest. At least in the case of conceptual methods, i.e., methods whose primary objects are concepts, the problem is usually solved by a change in the explicit knowledge state. The knowledge state is supposed to have raised a problem before the use of a method. A problem is usually a

[1] This is not the only model provided in our research. Another model was developed by Gahér and Marko in Gahér (2016), Gahér – Marko (2017). Their model is not discussed here.

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question that does not have an explicit answer within the initial knowledge state. The use of a method changes the knowledge state in order to solve the problem. In other words, the new knowledge state should contain an explicit answer to the problem or at least provide enough means to acquire the answer. An analytic method is suggested to be a method that does not cross some type of logical closure of the initial explicit knowledge state.

During our research of supposed analytic methods (such as explication, definition, conceptual analysis, idealization and abstraction, all taken as sets of instructions[3]), we discovered an inherent underdetermination of these methods. In their ideal form, these methods often contain a point at which choice is required. The outcome of this choice cannot be predicted and is not determined. Therefore, the results of the use of these methods can vary. Thus it is not sound to consider methods as functions over knowledge states but rather as relations over the set.

One of the presuppositions of our model of a scientific analytic method seems to be that it enriches the ontology of the knowledge state. In this paper, I want to elaborate on the idea of analytic methods that tend to do the opposite – they reduce the ontology of the state of the knowledge. If we model methods as relations over the knowledge space and we pick out a method as analytic according to the suggested definition, then we can consider whether its inverse relation over the space is a model of an analytic method as well.

The paper is structured into the following sections. In the first section, I briefly present those parts of the instructional model of scientific methods (IMSM hereafter) which are most relevant to the discussion in this paper. In the second section, I briefly recapitulate the definition of analytic method provided in our research. The third section provides a case study of a widely used method which should be considered analytic and stands as an example of

[3] For the provided models see Bielik (2015), Halas (2015) (2016a,b), Kosterec (2016b), Zouhar (2014) (2015a,b).

a method used to reduce the ontology of the knowledge state. The next section challenges some of the features of the instructional model – mainly the monotonicity of ontology. I also discuss whether monotonous ontology is a presupposition of the use of analytic methods. Based on the results of preceding sections, the fifth section provides an analysis of whether there is an inverse method for every analytic method. The paper ends with a brief conclusion.

1. The instructional model of method

We ventured on the way of developing a formal model of scientific methods in order to be able to exactly research and describe their interesting philosophical and methodological properties. One of the main research goals was to propose a non-trivial definition of analyticity of a method (more on this in the second section). First, we focused on the linguistic forms in which methods are usually provided to an agent. Based on the fact that methods are usually conveyed by means of imperative sentences, we went on to analyse the notion of an imperative. We stipulated that the role of the imperative to the agent is to urge the latter to change some state. Because we focused on methods used in science, we were mainly interested in methods providing changes in the state of knowledge. Every state of knowledge should have, firstly, a universe of objects which the knowledge is about. This knowledge is, secondly, grasped by particular conceptual tools. Finally, the state of knowledge is represented by a sum of propositions with an ascribed epistemic value.

The purpose of a method in general is to lead an agent to effect a desired change. In connection to the knowledge base, the motivation for the change is usually that the agent does not have enough information to solve some problem. In science, a problem is usually posed in the form of a question. Therefore, a scientific method is used to change a knowledge state of an agent from a point which raises questions to a different state in which those questions are answered.

Building on these intuitions, we analyzed particular relevant semantic properties of instructions which were considered as the semantic content of imperative sentences, at least for the purposes of analysis of scientific methods. Every instruction is taken as a relation among knowledge states. In other words, an instruction can be viewed as a set of steps between states of knowledge. This way of modeling the content of an instruction serves well to grasp another relevant feature of scientific methods. They usually do not consist of a single instruction, but are rather represented by an ordered set of instructions that follow one another and prescribe a more complex change of the knowledge state. Because we modeled instructions as relations over knowledge states, the combination of instructions into bigger wholes was quite straightforwardly modeled by the composition of such relations.

From a general point of view, a method is taken to change the knowledge state of an agent by changing at least one of the parts of his knowledge state (i.e., the universe of objects, the conceptual system or propositional knowledge). The initial state for using the method includes those parts of knowledge base which are in need of change as its *ontology*. The method then provides changes in the ontology either by some actual executive computational steps (derivative steps) or by adding some new postulates (postulate steps). The main part of the stipulated model of method I want to discuss further is the thesis that the ontology *after* using a method *always* contains the ontology *before* using the method as its subpart. In other words, the only changes modeled by the formal version of IMSM were based on enriching the knowledge state of the agent.

2. The definition of analytic method

While the model of method presented by IMSM enabled us to model the basic parts of methods and their composition, it did not contain a definition of analyticity. The main question for our research was: *What makes a scientific method analytic?* Nevertheless, we embarked on detailed case studies of known conceptual methods often considered as analytic throughout several fields such as philosophy, sociology or economics. Based on the material this research provided (usually an ordered set of instructions), I could roughly differentiate among three types of instructions that usually take part in a method: selective, executive and descriptive. Selective instructions urge an agent to pick one of several possible choices. Within a knowledge base, we can imagine an example when an agent makes a choice whether they will prove the validity of an argument by a direct or an indirect proof. The act prescribed by a selective instruction therefore does not provide any new information not already included in the knowledge state of the agent. Descriptive instructions urge an agent to make the results acquired by following the method publicly available. These steps, of course, also do not provide the agent with any new knowledge. The main type of instructions were executive instructions. They urge an agent to actively engage in a change of the state of knowledge by following the action prescribed by the instructions.

The definition of an analytic method provided was based on the intuition that analytic method should not lead to any new empirical information, but rather should help in decoding, finding or making explicit some information that had already been present implicitly in the state of knowledge. This could raise doubts whether there really can be an gain in information (see Ackermann 1992), but these questions are already settled (see Duží (2010), Duží – Jespersen (2013)).

An agent following the analytic method cannot find any information which is not entailed by the state of his initial knowledge. I defined a method as analytic if the executive instructions it contains do not make the agent cross the logical closure of his initial knowledge.

In the first two sections I described the state of research about the analytic scientific methods we have reached so far. In the following sections, I want to discuss some non-trivial properties of the described models, mainly the monotonous growth of parts of knowledge state prescribed by the formal model of IMSM. But first, let me present a case study which I will then refer to in my arguments.

3. A case study

This section is set to provide an example of an analytic method used to reduce the content of some part of the knowledge state. According to our model, it should reduce the universe, the conceptual scheme or the state of propositional knowledge. I consider the following scenario which often appears, e.g., in the analysis of the mathematical properties of acquired information or in the solving of mathematical equations. Solving a set of equations is a quite common problem.[4] As we all know from elementary school, when there is a set of equations, there are three possible scenarios. Either there is no solution, or there is exactly one solution, or there are infinitely many solutions. A necessary condition for a single solution to exist is that the number of linearly independent equations is equal to the number of variables. For example, consider the difference:

a) $1x + 2y = 6$

   $2x + 4y = 12$

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4 I will talk only about linear equations.

b) $1x + 2y = 6$

$4x - 1y = 6$

The equations under a) are not linearly independent – the second equation is the first multiplied by 2. Therefore, this set of equations has more than one solution. On the other hand, the equations under b) are linearly independent and the set has exactly one solution.

Now imagine we are in a state with 10 equations that contain the same two variables. Does this set of equations have exactly one solution, an infinite number of solutions, or none at all? There is a method to find out. It is called the *Gauss Elimination Method* (GEM). In short, this method reduces the set of equations containing the same variables into a set of linearly independent equations. By this we can find out whether the set of initial equations has an infinite number of solutions or at most one. If the number of linearly independent equations is lower than the number of independent variables, then the set of equations has an infinite number of solutions. The method is widely used and, as a purely mathematical method, it can easily be considered analytic.

Let us analyse the use of this method using intuitions behind IMSM. I think it can be considered an example of a method that reduces the ontology of the knowledge state of the researcher. It is enough to consider the initial set of 10 equations as the universe of objects of the initial state. This state does not contain any information on the number of solutions to the set of equations. In other words, the set of propositional knowledge does not contain a proposition containing such information. Now, it is safe to say that at least for some initial states, the set of equations will not contain only linearly independent equations. But after using GEM, the set of equations always contains only linearly independent equations. Therefore, the number of the equations is reduced, along with the whole ontology of the knowledge state. Of course, we can say that the amount of information has increased, because now the set of propositional knowledge contains the information that the ontology – the set of equations – contains only linearly independent equations. Therefore, we can consider GEM as an example of an analytic method which does not abide by the monotonicity of the ontology presupposed by IMSM – at least in the form it was presented. Based on this example, I shall suggest in the next section that the monotonicity condition on ontologies should be dropped from the instructional model of method.

4. Monotonicity challenged

In this section I want to discuss the following two questions which challenge the notion of monotonous growth of ontologies presupposed by IMSM:

a) *Does monotonicity of growth of ontologies in IMSM respect the intuitions about knowledge change provided by the use of scientific methods?*

The short answer to the first question is: No, it does not. I defend this claim using two examples. By doing so, I falsify the general claim that the ontology of a knowledge state (i.e., the universe of objects, the conceptual scheme, and propositional knowledge or their combinations) always grows or is at least monotonous in its growth every time an agent uses an analytic method.

The first example is inspired by Bielik's model of method of sampling (see. Bielik et al. 2014d). If we analyse it properly, we can see that the ontology before the use of the method need not be a subset of the ontology after the use of the method. The problem leading up to the use of the method of sampling is the need to acquire a simpler (but still representative) set of objects from the initial universe of objects. Whatever actions the method of sampling would require, the agent does not need to use the whole set of objects. A representative set suffices. Therefore the use of the method of sampling, although modeled by instructions according to IMSM, does not require the ontology of the initial state to be a proper subset of the ontology of the final state. In other words, although the model of the method of sampling is instructional, it does not presuppose the monotonous growth of the ontology. However, I do not claim that Bielik's model of the method of sampling does not respect the monotonicity required by IMSM. Rather, I argue that the monotonicity required by IMSM need not be present in some correct alternative model of the method of sampling. The second example of analytic method which does not presuppose the monotonous growth of the ontology of knowledge state was already presented in the previous section.

The two examples above make a strong case against the monotonous growth of ontology presupposed by IMSM. In general, the ontology of the initial knowledge state can be given by the universe of the state, the conceptual scheme of the state, and propositional knowledge of the state or combinations of these. The examples were cases against the monotonous growth if the ontology consisted of a universe of objects of the knowledge state. But I think one can also argue against the monotonous growth of other possible choices of ontology. In the case of propositional knowledge, consider a method which provides a falsification of some proposition. The result of the method changes the epistemic value ascribed to a proposition within the propositional knowledge of the final state of the method. IMSM is based on set-theoretic notions. If we compare the initial propositional knowledge and final propositional knowledge in the considered case, they are not in the relation subset – set. They are two sets with a non-empty intersection. And in the case when the conceptual scheme is picked as the ontology of an initial state, we can ask, for example, whether the method of explication presupposes a growth of ontology, i.e., of the conceptual scheme. This method is used to modify the conceptual scheme, not to enrich it. We use the method of explication to replace a concept which no longer plays an important theoretical role with some other concept. After applying the method of explication, the conceptual scheme of the theory no longer contains both concepts. These considerations weaken the need for the monotonous growth of ontology as an indispensible presupposition for any correct model of scientific method.

b) *Does the definition of analytic method require monotonous growth of ontologies of knowledge states?*

In short, it does not. The main condition of a method being analytic posed by our definition was that the main executive steps of the methods do not cross the logical closure of the initial state before the use of the method. Now consider the following. There are several different axiomatizations of propositional logic. For the discussion here it is important to acknowledge that although these axiomatizations are different, they all specify the same universe of tautologies of propositional calculus. Therefore, we can consider a method which would consist in finding a different axiomatization of propositional logic from the one that is currently at hand. The ontology of the initial state of the use of such method would consist of the axioms of the initial axiomatization. The ontology of the final state would consist of axioms of a different axiomatization. These two sets of axioms need not be in the relation subset – set. Nevertheless, they will be part of the same logical closure – namely, both will be within the universe of tautologies of propositional calculus. This is enough for the method to be considered analytic according to the definition. Therefore the definition of analytic method does not require the monotonous growth of the ontology of the knowledge states as presupposed by IMSM.

5. Inverse analytic methods

The main result of the preceding sections is that there is no reason for an analytic method to provide an expansion of the knowledge state in general. The sheer amount of methods which have some kind of reduction as a goal can be considered as a relevant empirical fact for this line of reasoning. In general, we do not use analytic methods solely for the purpose of extracting some new information which had been hidden within the context of the problematic knowledge state. We can also use them to reduce the amount of explicit but unnecessary information which is within the knowledge state. For example, to write an abstract of a paper is to enable the readers to get a quick overview of the content of the article without having to read the entire paper. But surely the abstract must not contain any information not contained in the paper itself. GEM, described in the third section, is another example of a reductive method.

Now, if methods are considered as relations on possible knowledge states of agents and the monotonicity of growth is no longer viewed as a necessary condition for any relation to be considered an analytic method, we can ask the following question: Is there an inverse analytic method for each analytic method? One can ask, though, whether this question is really important. The motivation behind it is that if the answer was positive, then the set of all analytic methods would be a group, which would open the whole field of abstract algebra and tools from within that field for methodological purposes.

The general answer is: No. Although there is a concept of inverse relation to any relation, we cannot assume that an agent who would follow a method and then its inverse would necessarily return back to where they started. The reason being that the composition of a relation with its inverse does not lead to an identity relation. It can easily be seen on the following example. Consider A, B to be possible knowledge states. Let us have two methods, which are inverses as relations:

M1: {<A, B>, <B, B>}

M2: {<B, A>, <B, B>}

The result of their composition is the following

M1 ∘ M2 : {<A, B>, <A, A>, <B, B>, <B, A>}

which is not an identity relation. It only contains the identity relation as its proper subpart. If an agent follows this method from point A, it does not necessarily lead them back to point A. Therefore, the set of analytic instructions cannot be considered a group.

But what about those analytic methods which are really just functions? In other words, not all analytic methods must lead an agent from one initial state to several possible states. Do these have inverses at least? In general, no. Only binary functions (i.e., injective and surjective) have inverses which after composition lead to identity. Although I cannot deny the existence of inverse analytic methods in general for at least a subpart of the universe of analytic methods, I can deny it for the analytic methods we presented a model of using IMSM: defining, explication, conceptual analysis, abstraction and idealisation, and so on. The main reason is that even if we strip them of non-deterministic steps (selective instructions), we cannot consider them to be one-to-one mappings on states of knowledge. The simple reason is that a single definition, explication, abstraction or conceptual analysis can be the solution to many different initial problems. And this is especially the case in field of philosophy, where a good definition can provide real advances in many debates. Consider, e.g., Kripke's famous definition of a rigid designator.

6. Conclusion

This paper set forth to analyse the dependence of the notion of analytic method on the monotonous growth of states of knowledge. Using several examples, I argued against such dependence. This opened the plane for analysis of the possible existence of inverses of analytic methods. Although I did not provide any proof against the existence of inverses for at least some analytic methods, I argued against the existence of such methods for analytic methods in general. Importantly, the methods viewed as central to conceptual research, such as conceptual analysis or explication, do not have inverse analytic methods.

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